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Pressures from flowing granular solids in silos

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THE ROYAL SOCIETY

Pressures from flowing granular solids in silos

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Large displacement flows in granular solids occur whenever silos are discharged. Measurements of pressures during flow, combined with visual observations of flow patterns and control tests on the ensiled solids, have revealed several phenomena contributing to pressure variations during flow. Not only are the pressure variations time dependent within a single silo, but significant systematic differences are found between one silo and another, even when the two are superficially identical and contain similar ensiled materials. One serious outcome is that silo pressures are quite unsymmetrical even in symmetrical silos, and this is a most dangerous phenomenon for the safety of the silo structure.

The loss of symmetry can largely be traced to inhomogeneity and anisotropy in the ensiled material, developed by the initial packing during filling. Thus, unless the mechanism of packing is understood, the mechanics of these solids cannot be modelled. The sensitivity of granular materials to stress history and the role of geometrically imperfect boundaries present other complications in interpreting observations and understanding the mechanics of silo pressures and flow regimes.

No current theory of silo loads covers these phenomena, which themselves are only illustrative examples of current shortcomings. More comprehensive constitutive models are needed for application to silos and large-scale granular solids flows.

Keywords: packing of particles; anisotropy; inhomogeneity; symmetry; scale errors

1. Introduction

Research on silos is a challenge to all researchers, although the starting point seems rather simple: a container is filled from the top with a granular solid and discharged through an outlet in the bottom. This has been done for centuries to store grain from one harvest until the next.

A simple theory was devised by Janssen (1895) late last century. It explains how the pressure increases with depth and as a result of wall friction, which transfers vertical load from the stored material to induce compression forces in the wall. With increasing depth, the condition is gradually approached in which all the load from further heads of the solid is transferred to the wall so that the pressure level does not increase; an asymptotic condition is approached (see figure 1). Only the compression force in the wall still increases. The horizontal pressure in the lower part of such a silo is only a small fraction of the corresponding pressure in a similar water container.

An understanding of this phenomenon led to a simple structural system for silos: a tall reinforced-concrete cylinder with a circular planform, which needs only a moderate amount of horizontal steel reinforcement to resist the horizontal tensile forces: the concrete itself can easily carry the vertical compression forces.

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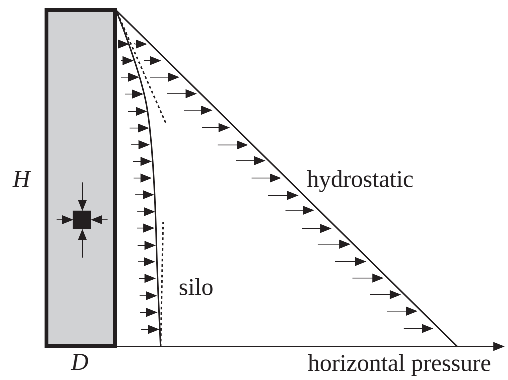

Figure 1. Silo pressure approaches an asymptotic level which for silos with circular planform is $\gamma D/4\mu$, where γ is the unit bulk weight, D is the silo diameter, and μ is the wall friction coefficient. Due to the internal strength of particulate materials, the horizontal pressure in a silo is only about half of the vertical pressure: with the same bulk unit weight, the horizontal pressure just below the surface in a tank increases about twice as fast with depth as in a silo.

During the 1950s and 1960s, economic development led to larger and larger silos. At the same time silos became popular structures in process engineering and in bulk solids handling, which meant that silos were used to store materials that had not previously been stored in large quantities. The simple ideas which were adequate for small silos were assumed to apply equally to these larger structures.

The outcome of that extrapolation was serious damage to many silo structures. This showed that although hydrostatic pressures in fluids are well defined and predictable, that is not the case for granular solids pressures, especially during discharge.

It became obvious that the classical description was an unacceptable oversimplification and a search started for the parameters which most influence the behaviour. This paper may be seen as a summary of the key findings of this search for new knowledge. The goal was to guarantee the structural integrity of the silos. The paper concentrates on phenomena which have been studied by the author or research groups with whom he has been closely associated. These phenomena are much concerned with concrete silos filled with grain or fly ash (see figure 2).

Most of their observations were made in either large concrete silos, a 5 m tall model in the laboratory, or in small centrifuge models. The measuring devices were pressure cells, strain gauges, and units for measuring changes in crack openings in concrete walls (see Askegaard *et al*. 1971; Askegaard & Munch-Andersen 1985; Askegaard & Nielsen 1986; Askegaard 1995).

The phenomena were studied by using a scientific approach involving the following steps: observe, understand, generalize and predict. However, the phenomena have been so complicated that generalization has often proved difficult and sometimes impossible, and the formulation of theories which quantitatively predict the phenomena is still considered a formidable challenge (Rotter 1998).

While the paper does not attempt to give a complete list of all the phenomena which may occur in silos, many of the phenomena which are described relate to the general fundamental behaviour of granular materials and therefore have general interest for researchers into granular materials.

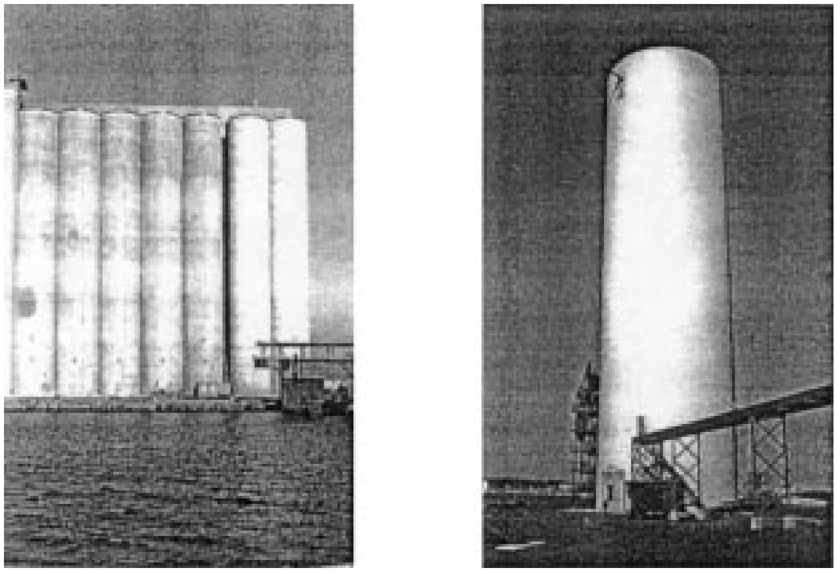

Figure 2. A silo battery for grain at Falkenberg, Sweden (height 50 m, 8 m diameter for each tube) and a fly ash silo at Kalundborg, Denmark (16 m diameter, storage height 28 m, total height 50 m).

2. The history of a particle in a silo

A closer look at the history of a particle which passes through a silo reveals several more aspects of silo phenomena than those noted in the Introduction (see figure 3). The complete life cycle of granular material (Muir Wood, this issue) can be seen in a single passage through the silo:

During filling the particle passes through the inlet, loses contact with other particles, and *falls through the air* until

it *impacts on stationary particles* at the surface of the stored material, possibly after *impact with the wall*, after which it may

bounce, or slide down the surface, or float in suspension (powders) on the surface, where it may be

hit by particles arriving later, until it finally

embeds as a member of a stacked particle assembly.

As filling continues the particle participates in a *consolidation*, which may involve varying interstitial air pressure as a result of air entrapped during filling.

During this process, contact forces vary and *contacts between a particle and its neighbours may be rearranged*, especially if the particle is sitting near or in contact with the wall.

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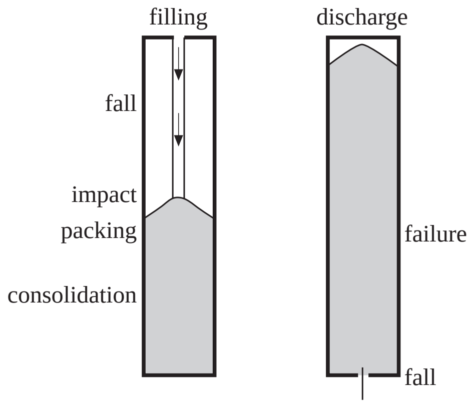

Figure 3. Overview of events in a silo.

> During discharge, the particle follows a certain track determined by the flow pattern in the silo. Here, it passes through zones in which it may keep in contact with neighbouring particles (rigid body movements) and/or pass zones where those contacts are rearranged due to failure either in large groups of particles or in narrow zones or planes. During these periods, *contact forces may vary dramatically.*
>
> Finally, the particle approaches the outlet, relaxes its contacts with other particles and *leaves the silo* in free fall (if no discharge aid is used).

It is obvious that these phenomena cannot be described by using only the few parameters that were introduced in the classical theory explained in the Introduction. It is less obvious whether all the above phenomena are important in even a basic description, or under what circumstances and to what extent each of these phenomena might play a significant role in the overall safety level of the silo.

3. Observed silo phenomena

Experiments in different types of silo at full scale as well as in models have led to a series of observations of phenomena associated with storage of granular materials.

(*a*) *Filling and particle stacking modes*

The filling method was not recognized as important until tests on full-scale Swedish grain silos (Askegaard & Nielsen 1976) revealed a major influence. Measurements of crack openings in these damaged silos indicated that a circular silo with central outlet could develop significantly unsymmetrical pressure distributions both during filling and on discharge. This was confirmed by pressure-cell measurements (Hartlén *et al*. 1984), and led to a hypothesis about the influence of the particle packing structure caused by the filling method. Figure 4 shows four different packing phenomena which have been observed.

Landslides were observed on the sloping surface during filling with barley, but with wheat a different mode, here termed *cone squashing*, occurred during filling (Nielsen *et al*. 1982; Nielsen 1983).

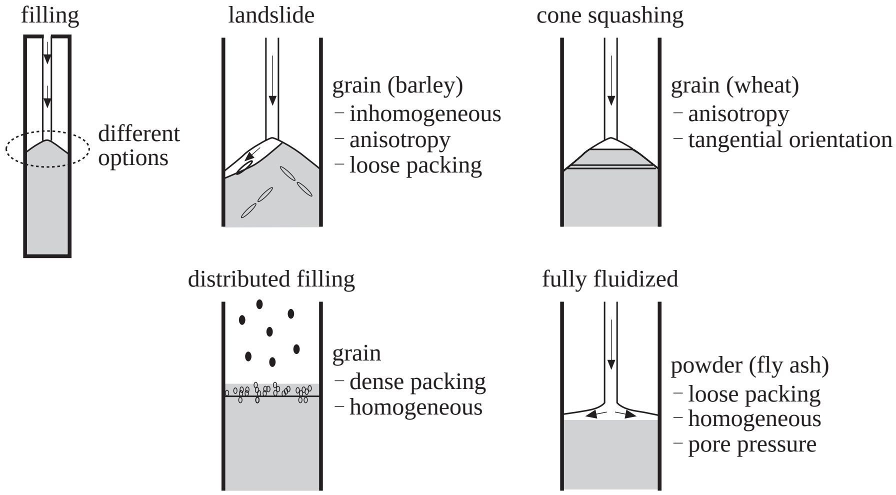

Figure 4. Different packing modes associated with filling of a silo.

Landslides occur because the slope increases until it becomes unstable and a tongue of material cascades down the surface in one direction. A little later a similar landslide occurs in another direction. As a consequence, the orientation of some of the particles is left in the direction of steepest slope (inclined, radial). The overall result is an ensiled material that can be characterized as inhomogeneous, anisotropic and with a low density. Because the landslide mechanism is intermittent, some irregularity (stochastic element) is embedded in the stacking.

Cone squashing is the result of pressure from the impact of incoming particles, which causes a plastification of the whole top section of particles. The top slowly and continuously squashes in a deformation mode where particles gradually move outwards, stretching in the peripheral direction and thus leaves the particles with a peripheral orientation. The overall result is an anisotropic ensiled material, but with the anisotropy more systematic and homogeneous than that caused by landslides and with quite different orientations.

In some tests *distributed filling* (Nielsen & Andersen 1982*b*; Munch-Andersen & Nielsen 1990) was arranged to achieve a homogeneous packing structure. Such a packing is dense. Distributed filling has been reported to produce a 4% higher unit weight in barley than stream filling.

Self fluidization has been observed on the surface of a silo during filling with fly ash (Nielsen 1984*a*). As a result of fall and impact, so much air is mixed with the fly ash that the mixture distributes at the surface like a liquid. This results in a homogeneous and loose stacking, which may build up interstitial pore pressure if the filling rate is high enough.

Grain silos in Sweden have often been constructed in blocks with a single conveyor above two rows of silos which are filled though inclined inlets. Grain particles enter with a horizontal velocity and strike the opposite wall. They are reflected from the wall at about mid-height and are spread so that distributed filling seems to be a fair description of the situation near the bottom. At mid-height, stream filling is a better description, but with the top of the free surface near to the wall, so that the anisotropy is built up with planar symmetry as indicated in figure 5. At a higher

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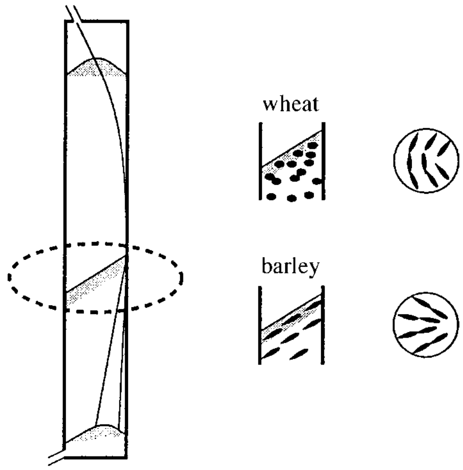

Figure 5. Different stacking situations in a Swedish grain silo with inclined inlet.

level notional rotational symmetry is again seen. The solid found in such a silo cannot be expected to exert symmetric pressures on the wall even in circular silo cells. A model for the complex stacking found in such silos is given in Nielsen (1983). Systematic studies of different filling methods and their influence on pressure distributions have been done by Munch-Andersen & Nielsen (1989, 1990) and Munch-Andersen & Askegaard (1993).

To sum up packing structures associated with filling processes: filling controls density, homogeneity, anisotropy and the interstitial pore pressure in the stored material and thus its strength and stiffness. The phenomena related to the filling arrangement are therefore an important part of the silo problem and they must be predicted if realistic descriptions of silo loads are to be achieved.

(*b*) *Pressure redistributions during storage*

A silo filled with barley was monitored during a period of four and a half months to explore the influence of grain consolidation on the pressure level (figure 6), which shows the pressure observed by one pressure cell over the first 28 days (filling occurred during the first 2 days) and over the last 37 days (discharge occurred during the last 2 days).

The pressure curve during storage exhibits some characteristic peaks. They have been ascribed to a relaxation mechanism in grain. After some relaxation, the lateral pressure is reduced to such an extent that the wall friction is no longer able to support the stored material and small settlements suddenly occur. As the settlement occurs, the horizontal pressure rises, the wall shears increase and equilibrium is re-established.

Towards the end of the storage period, these variations become smoother and can be correlated to changes in outdoor temperature, which causes wall movements inwards and outwards. In a steel silo the temperature of the wall may change much faster than that of the solid, so that relaxation cannot produce pressure variations to the same extent. Several brittle fracture failures due to sudden temperature drop have been reported (Rotter 1986).

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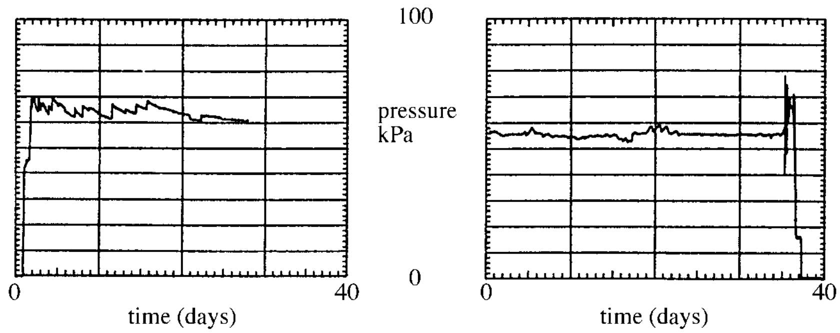

Figure 6. Pressure versus time during a four-and-a-half-month storage of barley in a silo.

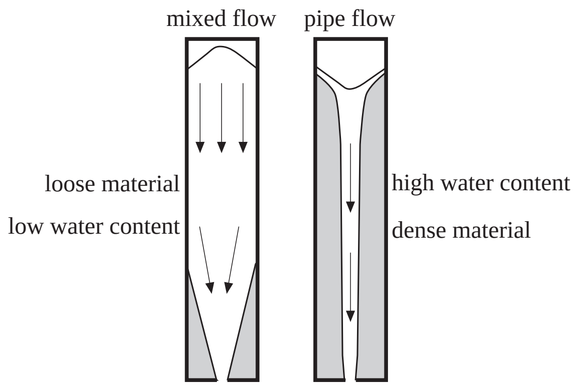

Figure 7. Flow patterns.

(*c*) *Shift of flow pattern*

Figure 7 illustrates two flow patterns which have been observed in grain silos: mixed flow and pipe flow. The two flow patterns can be distinguished easily by visual observations from above.

In pipe flow, particles start moving towards the centre of the surface shortly after the onset of discharge. At the centre, the particles move down through an internal pipe in the stored material. Wall pressure cells show only small changes if the discharge opening is central and the pipe does not touch the wall.

In mixed flow the shape of the surface is unchanged for the first period of flow during which the cylindrical part of the stored material moves as a single body. When the height of that body is of the order of half the silo diameter, a cave-in occurs and the flow pattern changes into a pipe flow. Mixed flow is associated with large masses in motion and considerable redistribution of wall loads takes place during discharge. If the silo bottom is a steep smooth cone, all the stored material moves during discharge and the flow pattern is called mass flow.

It has been observed that the same silo with the same type of stored material may operate in one flow mode under some circumstances and in another under slightly different circumstances. Stream filling of barley with a moisture content of 15% gave pipe flow while a similar barley with a moisture content of 12% under the same conditions gave mixed flow (Askegaard *et al*. 1971). Both levels of moisture content are considered to be relatively dry and acceptable for long-term storage. Changing

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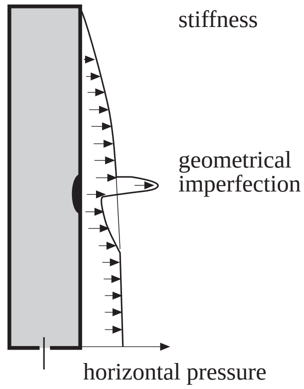

Figure 8. Wall imperfection with an indication of the possible redistribution in wall pressure.

the filling arrangement may also be sufficient to cause the change in flow pattern. Distributed filling of barley gave pipe flow while eccentric stream filling gave mixed flow (Nielsen & Andersen 1982*b*). Systematic studies have been done by Munch-Andersen & Nielsen (1990) and Munch-Andersen & Askegaard (1993).

These observations are taken as evidence that the shift from pipe flow to mixed flow takes place as an on/off phenomenon in an unstable regime, where small changes of secondary parameters may trigger this shift. The shift has considerable consequences for the pressure distribution and thus for the safety level of the silo structure.

(*d*) *Pressure distribution and geometrical wall geometry*

The stiffness of the stored material was not included in the traditional list of silo parameters, but experiments have shown that the solid has a considerable pressure redistribution capacity due to its stiffness (Askegaard *et al*. 1971).

Small irregularities in the silo wall produced by mounting a pressure cell can give rise to local alterations in the pressure on the wall, particularly during the discharge process. A change in the pressure of 50% was found when the surface of the pressure cell was changed from lying parallel with the wall to lying at an angle of only 1° to the wall in the vertical direction.

Similarly, an alteration of the geometry of the interior surface of the silo in the form of an artificial bulge (a projection of 6 mm over a diameter of 1000 mm) proved (especially during discharge) to cause considerable alterations in the internal forces in the concrete wall of a 4.5 m diameter silo (see figure 8) (Nielsen 1972).

The practical implication is that normal construction irregularities cause unsymmetrical loadings in notionally symmetrical silos. For research it means that wall pressures are difficult to measure, and since perfect silos in this respect scarcely exist, theories which assume perfect geometry cannot be verified.

Similar load redistributions take place when wall deflections give rise to deformations of the stored materials: a structure–solid interaction phenomenon.

(*e*) *Boundary layer in rough silos*

With mixed flow, a block of stored material slides down the wall. However, if the wall is rough, the particles nearest to the wall do not move at the same velocity

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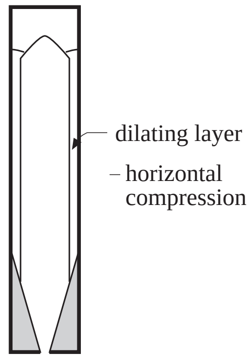

Figure 9. Boundary layer.

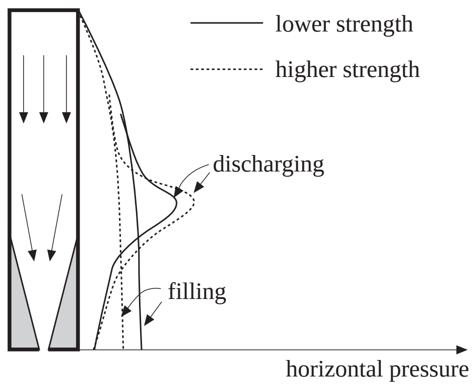

Figure 10. Redistribution potential. Stronger stored material leads to lower filling loads but may give rise to higher discharge loads than weaker materials.

as the remaining part of the material (see figure 9), producing a boundary layer of slower particles. This causes a small heap of grain to develop at the surface around the periphery (Askegaard *et al*. 1971). The boundary layer, including its dilation, has been studied in detail by Munch-Andersen (1987). He concluded that the width of the boundary layer is related to the particle diameter rather than the silo diameter. Especially with dense packings, dilation of the boundary layer is needed to permit vertical movement. In small silos this dilation causes a horizontal strain that changes the stress field completely.

(*f*) *Pressure distribution and strength of the stored material*

The strength of a granular material was represented in the classical theory by the lateral pressure ratio (horizontal to vertical) and the wall friction coefficient (rough walls). Where the solid has higher strength, the horizontal loads on silo walls become smaller. Tests have confirmed this under storage conditions. However, during discharge (Nielsen & Andersen 1982*a*; Munch-Andersen & Nielsen 1990), strong solids cause larger pressures than weak solids in silos where the discharge flow pattern involves forced deformation (inserts, bulges, etc.).

This may be reformulated in terms of a stress redistribution potential: the stronger the material, the bigger the possible wall pressure deviation from the average (see

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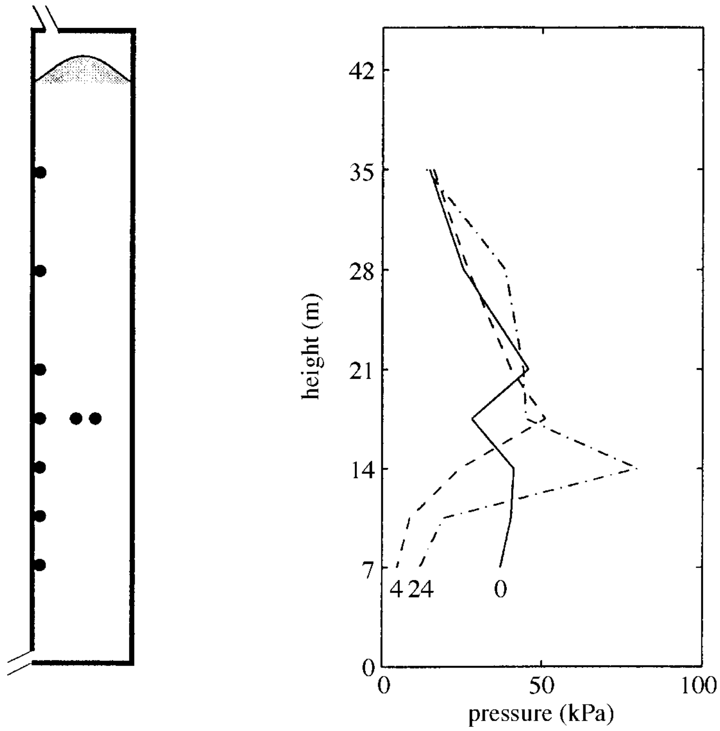

Figure 11. Pressure redistribution with time. The figure shows the pattern of pressure cells on a silo wall and the observed pressure distributions down the generatrix above the eccentric outlet. The distribution marked 0 shows the pressure at rest and the other two show pressure distributions at 4 min and 24 min after the start of discharge. The two additional pressure cell positions indicated at the height of 17.5 m correspond to observations shown in figure 12.

figure 10). Discharge overpressures should therefore be taken into account in design: a suitable method is by applying positive or negative patch loads (depending on the material stiffness) rather than by using a general overpressure factor, which has commonly been done in design standards.

Fine powders and many other materials may develop cohesion in silos. They become relatively stronger, causing a bigger redistribution potential, which has led to structural disasters.

(*g*) *Pressure variation with time*

While the pressure at a given position does not change very much during storage (see figure 6), substantial variations can be observed during discharge, especially in silos with mixed flow. Figure 11 gives an example (Nielsen & Andersen 1982*a*), where the increase 24 min after the start of discharge at a certain position is about 100% and the decrease at another time and position is about 90%. The pressures are quasi-static: they are locked in and remain if the discharge is stopped. The example also indicates that silos may have to withstand many different loading cases during one filling and discharge.

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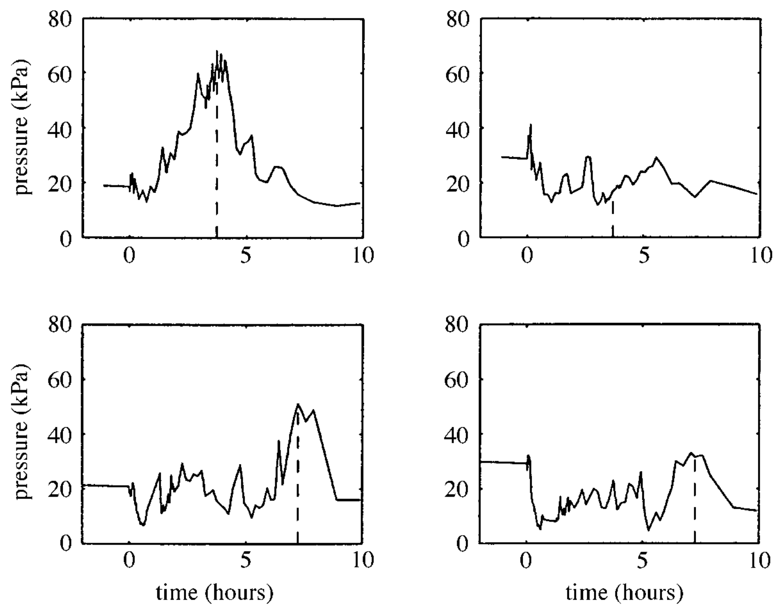

Figure 12. The figures show pressure versus time at two positions in the Karpalund Silo, Sweden (Hartlén *et al*. 1984) in a test series on barley. The upper pair of figures are from one test and the lower pair from another. Discharge starts at time zero. The two positions are separated by only 500 mm at the height of 17.5 m indicated in figure 11.

(*h*) *Symmetry/repeatability of tests*

Figure 12 shows pressure versus time on two pressure cells in two tests of the same series on barley. The upper pair of figures are from one test and the lower pair from the other. Several observations can be drawn from such recordings:

> Fluctuations in pressure are seen with either short or long duration (of the order of seconds to minutes).
>
> Repeatability of tests is low. The maximum pressure and the instant at which it occurs are different from test to test.
>
> Very big pressure gradients may occur and remain stable for a long time, for about an hour (see the dashed vertical lines in figure 12). Gradients of about 200% of the storage pressure level over a distance of only 500 mm are seen. A stress discontinuity may be an appropriate interpretation of this phenomenon.
>
> Readings from individual pressure cells should be interpreted with caution, because they may represent pressures on large parts of the wall and be of importance to the structural integrity, or they may represent more local effects of less significance.
>
> Symmetrical pressure distributions are not seen during discharge in mixed flow, even if the conditions are notionally symmetrical.

To some extent the pressure redistribution during discharge may be considered a stochastic process. The fluctuations and the lack of symmetry may arise from a global instability triggered by small local events, comparable with chaotic behaviour.

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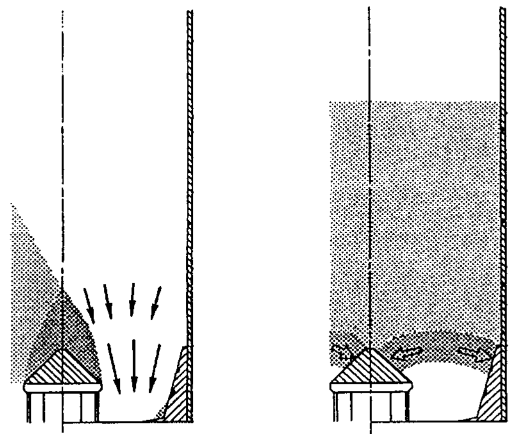

Figure 13. Fly ash silo, 16 m diameter (Nielsen 1984*a*). The silo is discharged by sector-wise aeration of the slightly inclined bottom. Two types of flow pattern exist. In the left-hand figure, the flow zone only develops above the aerated part of the bottom. In the right-hand figure, a rotationally symmetric condition occurs because the stored material near the bottom is fully fluidized: it matters little which part of the bottom is aerated.

The above observations can be summarized in the following statement on the nature of discharge pressure in grain silos. During discharge, there seems to be an interplay between pressure release from below and compression or forced deformation from above, especially in densely packed granular solids. In silos with mixed flow, the moving body of material may be split by rupture on planes with stress discontinuities into different bodies, each finding their individual way downwards in ever changing shapes, creating stress history, anisotropy and inhomogeneity in addition to the inheritance of filling.

Tests in a fly ash silo, however, have revealed different characteristics (Nielsen 1984*a*). Here pressure gradients and fluctuations are small and a pressure cell observation seems to represent a much larger area than in a grain silo. This has been explained by the looser packing which reduces the stiffness and hence the influence of wall geometric imperfections, and leads to rupture patterns developing in zones rather than on discrete planes, so that pressure gradients are smaller.

(*i*) *Dynamics*

Figure 13 shows two different flow patterns in a fly ash silo. The flow pattern with the fully fluidized bottom zone (to the right) was associated with a high aeration pressure supporting a bridge of stored material which could suddenly break down. A loud noise was heard and the whole silo shook. In the left figure the intended operating condition is shown with sector-wise pipe flow. This was achieved after a reduction in the aeration pressure. In this condition, little potential energy was built up and no dynamic phenomena were experienced.

Another more periodic form of dynamic pressure variation was studied by Nielsen & Ruckenbrod (1988).

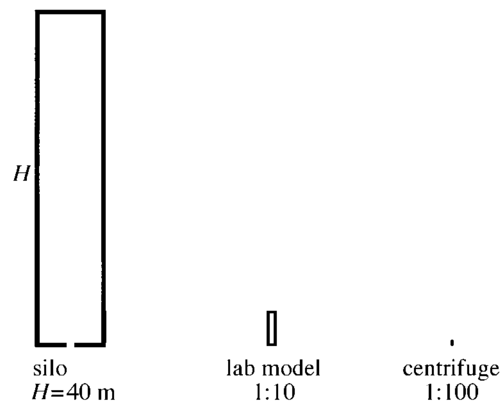

Figure 14. Different scales of silo. It is a challenge to predict loads in large silos based on model tests.

(j) *The challenge of modelling*

Many interesting phenomena are associated with scale models. Centrifuge modelling is valuable from a theoretical point of view (see Nielsen 1977). If a continuum approach is adopted, and the field of gravity is raised by the same factor as the geometric scale is reduced, the same stresses and strains occur at similar points, and no assumptions are needed about the stress dependence of the constitutive relations, and no scaling of particles is required. This is very important because the complexity of these relations may be responsible for many of the phenomena not covered by the traditional continuum description.

Scaling problems arise with phenomena which are related to particle size: here the continuum approach is not valid, and scaled particles may be needed. For some materials (e.g. grains) this is impractical (Nielsen 1977), while for crushed materials it may appear viable, but leads to new problems associated with the different behaviour of powders.

In practice, centrifuge models are limited in size due to the experimental technique (see figure 14). A centrifuge model might be about two orders of magnitude smaller than full scale. A special phenomenon associated with centrifuge testing is Coriolis forces (Nielsen 1977, 1984*b*). Figure 15 shows how the particle trajectory may be strongly curved in the model.

Having described the influence of the filling method on the overall behaviour of the stored material it is clear that serious scale errors may occur with coarse grained materials. Conflicting time-scales for pore pressure, creep and inertia forces may also lead to scale errors whether the particles are scaled or not (Nielsen 1977).

For tests in the natural field of gravity, the main problem is that stresses scale almost proportionally to the geometric scale, and if cohesion does not scale in the same way, only purely frictional materials can be modelled.

Many different types of model tests have been performed (see Nielsen & Askegaard 1977; Nielsen & Kristiansen 1980; Nielsen *et al*. 1982; Munch-Andersen 1983; Munch-Andersen & Nielsen 1984, 1986, 1990; Kristiansen *et al*. 1988; Munch-Andersen *et al*. 1992; Munch-Andersen & Askegaard 1993). The overall assessment of these tests is that the main uncertainties about the validity of the results are those listed in table 1.

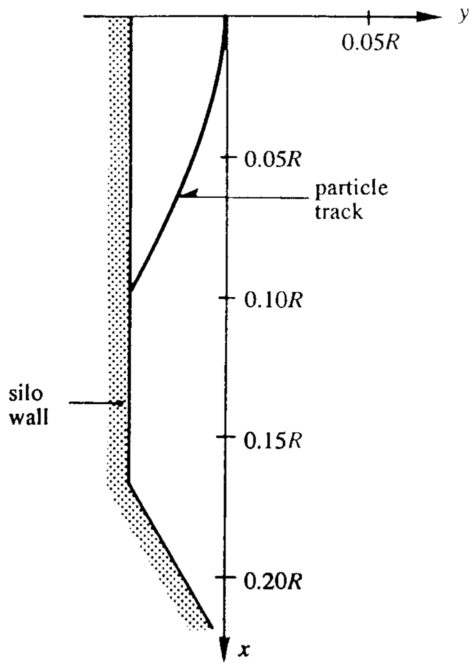

Figure 15. Particle trajectory in a centrifuge model which has a height of about 20% of the centrifuge arm, R.

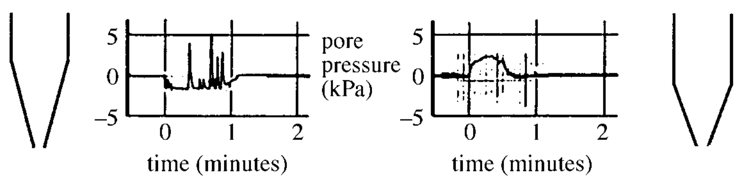

Figure 16. Pore pressure measured in lime powder in the upper part of the hopper. In the left-hand figure, the pore pressure is negative in a steep hopper in which all the material in the cone fails with dilation (mass flow). In the right-hand figure the corresponding pore pressure is shown in a less steep hopper with pipe flow, where loose material from the surface enters the pipe and passes through the hopper under increasing (low) pressure: the compaction gives an additional pore pressure (Kristiansen *et al*. 1988).

Table 1. *Main reasons for uncertainty about validity of tests*
(Different phenomena play different roles at different scales.)

solid	silo	laboratory model	centrifuge model
coarse	imperfections	imperfections	imperfections
		boundary layer	model particles
			filling
powder		cohesion	filling
		pore pressure	pore pressure

An example of phenomena studied in a centrifuge model is seen in figure 16, where the influence of dilatancy and compaction on pore pressure during powder flow is illustrated.

4. Challenges for the future

The descriptions in the previous section have demonstrated how the classical theory is unable to predict many important phenomena that take place in silos. However, these phenomena can only be taken as examples of a much larger variety of phenomena which have been discovered. Further information is available from a state-of-the-art report, which is a result of a Concerted Action project with participation of most European silo specialists and funded by the European Commission (see Brown & Nielsen 1998).

The nature of silo pressures is so complex that it is unrealistic to expect accurate theoretical predictions in the near future, and the search for new approaches to ensure structural safety, which has been in progress for some time, must therefore continue. In the absence of an adequate theory for the load consequences of anisotropy, inhomogeneity, wall imperfections, wall deflections, etc., load models with free patch loads have been introduced into loading codes to produce safe but conservative designs. These patch loads may also cover the interaction phenomenon where wall deflections, in combination with the stiffness of the stored material, cause pressure redistributions (Nielsen *et al*. 1992).

Rotter *et al*. (1986) analysed the need for different load models to be used for concrete and steel structures, due to the different failure modes they are likely to experience. Pham *et al*. (1986) and Munch-Andersen (1988) introduced a statistical approach, and Nielsen & Kolymbas (1988) introduced simplified methods to arrive at more realistic physical parameters to be introduced in load models. Some of these ideas have been introduced into load models in the Eurocodes (Nielsen *et al*. 1992).

During the past 50 years, the knowledge of particulate materials behaviour in silos has increased greatly, but the situation has been and remains that the more we learn, the more we find we do not understand. Many key questions are still to be answered.

> How relevant is the continuum mechanics approach to the description of silo phenomena? We have plenty of descriptions, and plenty of theories, but can they predict the packing structure (as a result of filling) that controls stiffness, strength, anisotropy, and inhomogeneity?
>
> What controls the shift between mixed and pipe flow, and the width of flow channels in pipe flow silos?
>
> Can we improve the constitutive equations? The influence of packing and of the stress–strain history is very important for silo phenomena. In the search for improved descriptions we may have to join researchers from the fields of soil mechanics and physics.
>
> Can computer simulations based on either continuum mechanics or single-particle methods be improved to give realistic answers? A single universal simulation program may not be a realistic aim for a problem with so many influential parameters which are still poorly understood. A better approach

may be to develop smaller programs, each devoted to a limited class of silo problem. It seems already possible to conduct interesting parametric studies, but predictions for practical design seem still some years ahead (Rotter 1998).

Will we ever be able to predict the spontaneous development of rupture planes by using only deterministic approaches?

Can we improve experimental techniques to give more reliable and complete observations, especially concerning the interior of the bulk of granular solid.

In conclusion, from a simple problem with few parameters, the design of silos has become a complicated one comprising many phenomena, most of which are also seen in other research areas. This suggests that a multidisciplinary cooperation could be valuable. Identified future needs are the following.

Better constitutive models (anisotropy, stress–strain history, etc.).

Better simulations to cover the most important phenomena.

Experiments for verification of simulation programs.

Load models that are simple, realistic and conservative.

The work referred to has been conducted in active cooperation with researchers from the following institutions: Danish Building Research Institute, Technical University of Denmark, University of Edinburgh, University of Karlsruhe, Brunel University, CSIRO (Melbourne), University of Sydney and University of Lund.

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